Some Infinite Products of Ramanujan Type

Ayşe Alaca, Şaban Alaca, and Kenneth S. Williams

Abstract. In his "lost" notebook, Ramanujan stated two results, which are equivalent to the identities

\[ \prod_{n=1}^{\infty} \frac{1 - q^n}{1 - q^{5n}} = 1 - 5 \sum_{n=1}^{\infty} \left( \sum_{d|n} \left( \frac{5}{n} \right) d \right) q^n \]

and

\[ q \prod_{n=1}^{\infty} \frac{1 - q^{5n}}{1 - q^n} = \sum_{n=1}^{\infty} \left( \sum_{d|n} \left( \frac{5}{n/d} \right) d \right) q^n. \]

We give several more identities of this type.