Two Volume Product Inequalities and Their Applications

Dedicated to Ted Bisztriczky, on his sixtieth birthday.

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Abstract. Let $K \subset \mathbb{R}^{n+1}$ be a convex body of class $C^2$ with everywhere positive Gauss curvature. We show that there exists a positive number $\delta(K)$ such that for any $\delta \in (0, \delta(K))$ we have $\text{Vol}(K_\delta) \cdot \text{Vol}((K_\delta)^*) \geq \text{Vol}(K) \cdot \text{Vol}(K^*) \geq \text{Vol}(K^\delta) \cdot \text{Vol}((K^\delta)^*)$, where $K_\delta$, $K^\delta$ and $K^*$ stand for the convex floating body, the illumination body, and the polar of $K$, respectively. We derive a few consequences of these inequalities.

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