Transversals with Residue in Moderately Overlapping $T(k)$-Families of Translates

Dedicated to Ted Bisztriczky, on his sixtieth birthday.

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Abstract. Let $K$ denote an oval, a centrally symmetric compact convex domain with non-empty interior. A family of translates of $K$ is said to have property $T(k)$ if for every subset of at most $k$ translates there exists a common line transversal intersecting all of them. The integer $k$ is the stabbing level of the family. Two translates $K_i = K + c_i$ and $K_j = K + c_j$ are said to be $\sigma$-disjoint if $\sigma K + c_i$ and $\sigma K + c_j$ are disjoint. A recent Helly-type result claims that for every $\sigma > 0$ there exists an integer $k(\sigma)$ such that if a family of $\sigma$-disjoint unit diameter discs has property $T(k)|k \geq k(\sigma)$, then there exists a straight line meeting all members of the family. In the first part of the paper we give the extension of this theorem to translates of an oval $K$. The asymptotic behavior of $k(\sigma)$ for $\sigma \rightarrow 0$ is considered as well.

Katchalski and Lewis proved the existence of a constant $r$ such that for every pairwise disjoint family of translates of an oval $K$ with property $T(3)$ a straight line can be found meeting all but at most $r$ members of the family. In the second part of the paper $\sigma$-disjoint families of translates of $K$ are considered and the relation of $\sigma$ and the residue $r$ is investigated. The asymptotic behavior of $r(\sigma)$ for $\sigma \rightarrow 0$ is also discussed.

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