A Class of Cellulated Spheres with Non-Polytopal Symmetries

Dedicated to Ted Bisztriczky, on his sixtieth birthday.

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Abstract. We construct, for all $d \geq 4$, a cellulation of $S^{d-1}$. We prove that these cellulations cannot be polytopal with maximal combinatorial symmetry. Such non-realizability phenomenon was first described in dimension 4 by Bokowski, Ewald and Kleinschmidt, and, to the knowledge of the author, until now there have not been any known examples in higher dimensions. As a starting point for the construction, we introduce a new class of (Wythoffian) uniform polytopes, which we call duplexes. In proving our main result, we use some tools that we developed earlier while studying perfect polytopes. In particular, we prove perfectness of the duplexes; furthermore, we prove and make use of the perfectness of another new class of polytopes which we obtain by a variant of the so-called $E$-construction introduced by Eppstein, Kuperberg and Ziegler.

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