Dunford–Pettis Properties and Spaces of Operators

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Abstract. J. Elton used an application of Ramsey theory to show that if $X$ is an infinite dimensional Banach space, then $c_0$ embeds in $X$, $\ell_1$ embeds in $X$, or there is a subspace of $X$ that fails to have the Dunford–Pettis property. Bessaga and Pelczyński showed that if $c_0$ embeds in $X^*$, then $\ell_\infty$ embeds in $X^*$. Emmanuele and John showed that if $c_0$ embeds in $K(X,Y)$, then $K(X,Y)$ is not complemented in $L(X,Y)$. Classical results from Schauder basis theory are used in a study of Dunford–Pettis sets and strong Dunford–Pettis sets to extend each of the preceding theorems. The space $L_{w^*}(X^*,Y)$ of $w^*-w$ continuous operators is also studied.

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