On the Simple $\mathbb{Z}_2$-homotopy Types of Graph Complexes and Their Simple $\mathbb{Z}_2$-universality

Péter Csorba

Abstract. We prove that the neighborhood complex $N(G)$, the box complex $B(G)$, the homomorphism complex $\text{Hom}(K_2, G)$ and the Lovász complex $L(G)$ have the same simple $\mathbb{Z}_2$-homotopy type in the sense of Whitehead. We show that these graph complexes are simple $\mathbb{Z}_2$-universal.

Department of Mathematics, Middlesex College, The University of Western Ontario, London, Ontario N6A 5B7
e-mail: pcsorba@uwo.ca

Received by the editors May 26, 2006; revised February 6, 2007.
Supported by grants from NSERC and the Canada Research Chairs program.
AMS subject classification: Primary: 57Q10; secondary: 05C10, 55P10.
Keywords: graph complexes, simple $\mathbb{Z}_2$-homotopy, universality.