On Axiomatizability of Non-Commutative $L_p$-Spaces

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Abstract. It is shown that Schatten $p$-classes of operators between Hilbert spaces of different (infinite) dimensions have ultrapowers which are (completely) isometric to non-commutative $L_p$-spaces. On the other hand, these Schatten classes are not themselves isomorphic to non-commutative $L_p$ spaces. As a consequence, the class of non-commutative $L_p$-spaces is not axiomatizable in the first-order language developed by Henson and Iovino for normed space structures, neither in the signature of Banach spaces, nor in that of operator spaces. Other examples of the same phenomenon are presented that belong to the class of corners of non-commutative $L_p$-spaces. For $p = 1$ this last class, which is the same as the class of preduals of ternary rings of operators, is itself axiomatizable in the signature of operator spaces.

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