On the Image of Certain Extension Maps. I

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Abstract. Let $X$ be a smooth complex projective curve of genus $g \geq 1$. Let $\xi \in J^1(X)$ be a line bundle on $X$ of degree 1. Let $W = \text{Ext}^1(\xi^n, \xi^{-1})$ be the space of extensions of $\xi^n$ by $\xi^{-1}$. There is a rational map $D_\xi : G(n, W) \to SU_X(n + 1)$, where $G(n, W)$ is the Grassmannian variety of $n$-linear subspaces of $W$ and $SU_X(n + 1)$ is the moduli space of rank $n + 1$ semi-stable vector bundles on $X$ with trivial determinant. We prove that if $n = 2$, then $D_\xi$ is everywhere defined and is injective.