Global Injectivity of $C^1$ Maps of the Real Plane, Inseparable Leaves and the Palais–Smale Condition

C. Gutierrez, X. Jarque, J. Llibre and M. A. Teixeira

Abstract. We study two sufficient conditions that imply global injectivity for a $C^1$ map $X : \mathbb{R}^2 \to \mathbb{R}^2$ such that its Jacobian at any point of $\mathbb{R}^2$ is not zero. One is based on the notion of half-Reeb component and the other on the Palais–Smale condition. We improve the first condition using the notion of inseparable leaves. We provide a new proof of the sufficiency of the second condition. We prove that both conditions are not equivalent, more precisely we show that the Palais–Smale condition implies the nonexistence of inseparable leaves, but the converse is not true. Finally, we show that the Palais–Smale condition is not a necessary condition for the global injectivity of the map $X$. 

Received by the editors May 9, 2005; revised October 30, 2006.

The first author is partially supported by FAPESP and CNPq–BRAZIL grants 03/03107-9 and 306992/2003-5, respectively. The second author is partially supported by MCYT grants MTM2005-02139 and MTM2006-05489. The third author is partially supported by a DGICYT grant number MTM2005-06098-C02-01. Both second and third authors are also partially supported by the CIRIT grant number 2005SGR 00550. The fourth author is partially supported by a FAPESP–BRAZIL grant 10246–2. All authors are also supported by the joint project CAPES–MECD grant HBP2003–0017.

AMS subject classification: 34C35, 34H05.