The Values of Modular Functions and Modular Forms

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Abstract. Let \( \Gamma_0 \) be a Fuchsian group of the first kind of genus zero and \( \Gamma \) be a subgroup of \( \Gamma_0 \) of finite index of genus zero. We find universal recursive relations giving the \( q \)-series coefficients of \( j_0 \) by using those of the \( q_h \)-series of \( j \), where \( j \) is the canonical Hauptmodul for \( \Gamma \) and \( j_0 \) is a Hauptmodul for \( \Gamma_0 \) without zeros on the complex upper half plane \( \mathbb{H} \) (here \( q_\ell := e^{2\pi i \ell z} \)). We find universal recursive formulas for \( q \)-series coefficients of any modular form on \( \Gamma_0^\times(p) \) in terms of those of the canonical Hauptmodul \( j_0^\times \).