

# Unimodular Roots of Special Littlewood Polynomials

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*Abstract.* We call  $\alpha(z) = a_0 + a_1z + \cdots + a_{n-1}z^{n-1}$  a Littlewood polynomial if  $a_j = \pm 1$  for all  $j$ . We call  $\alpha(z)$  self-reciprocal if  $\alpha(z) = z^{n-1}\alpha(1/z)$ , and call  $\alpha(z)$  skewsymmetric if  $n = 2m + 1$  and  $a_{m+j} = (-1)^j a_{m-j}$  for all  $j$ . It has been observed that Littlewood polynomials with particularly high minimum modulus on the unit circle in  $\mathbb{C}$  tend to be skewsymmetric. In this paper, we prove that a skewsymmetric Littlewood polynomial cannot have any zeros on the unit circle, as well as providing a new proof of the known result that a self-reciprocal Littlewood polynomial must have a zero on the unit circle.

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