Toeplitz Algebras and Extensions of Irrational Rotation Algebras

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Abstract. For a given irrational number \( \theta \), we define Toeplitz operators with symbols in the irrational rotation algebra \( \mathcal{A}_\theta \), and we show that the \( C^* \)-algebra \( \mathcal{T}(\mathcal{A}_\theta) \) generated by these Toeplitz operators is an extension of \( \mathcal{A}_\theta \) by the algebra of compact operators. We then use these extensions to explicitly exhibit generators of the group \( KK^1(\mathcal{A}_\theta, \mathbb{C}) \). We also prove an index theorem for \( \mathcal{T}(\mathcal{A}_\theta) \) that generalizes the standard index theorem for Toeplitz operators on the circle.