Vector Fields and the Cohomology Ring of Toric Varieties

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Abstract. Let $X$ be a smooth complex projective variety with a holomorphic vector field with isolated zero set $Z$. From the results of Carrell and Lieberman there exists a filtration $F_0 \subset F_1 \subset \cdots$ of $A(Z)$, the ring of $\mathbb{C}$-valued functions on $Z$, such that $\text{Gr} A(Z) \cong H^*(X, \mathbb{C})$ as graded algebras. In this note, for a smooth projective toric variety and a vector field generated by the action of a 1-parameter subgroup of the torus, we work out this filtration. Our main result is an explicit connection between this filtration and the polytope algebra of $X$. 