

# Uniform Estimates of Ultraspherical Polynomials of Large Order

*In loving memory of mia zia, Lucia Brogi in tributi*

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*Abstract.* In this paper we prove the sharp inequality

$$|P_n^{(s)}(x)| \leq P_n^{(s)}(1) \left( |x|^n + \frac{n-1}{2s+1} (1 - |x|^n) \right),$$

where  $P_n^{(s)}(x)$  is the classical ultraspherical polynomial of degree  $n$  and order  $s \geq n \frac{1+\sqrt{5}}{4}$ . This inequality can be refined in  $[0, z_n^s]$  and  $[z_n^s, 1]$ , where  $z_n^s$  denotes the largest zero of  $P_n^{(s)}(x)$ .

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