

Uniform Estimates of Ultraspherical Polynomials of Large Order

In loving memory of mia zia, Lucia Brogi in tributi

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Abstract. In this paper we prove the sharp inequality

$$|P_n^{(s)}(x)| \leq P_n^{(s)}(1) \left(|x|^n + \frac{n-1}{2s+1} (1 - |x|^n) \right),$$

where $P_n^{(s)}(x)$ is the classical ultraspherical polynomial of degree n and order $s \geq n \frac{1+\sqrt{5}}{4}$. This inequality can be refined in $[0, z_n^s]$ and $[z_n^s, 1]$, where z_n^s denotes the largest zero of $P_n^{(s)}(x)$.

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