Estimates of Henstock-Kurzweil Poisson Integrals

Erik Talvila

Abstract. If \( f \) is a real-valued function on \([-\pi, \pi]\) that is Henstock-Kurzweil integrable, let \( u_r(\theta) \) be its Poisson integral. It is shown that \( \|u_r\|_p = o(1/(1 - r)) \) as \( r \to 1 \) and this estimate is sharp for \( 1 \leq p \leq \infty \). If \( \mu \) is a finite Borel measure and \( u_r(\theta) \) is its Poisson integral then for each \( 1 \leq p \leq \infty \) the estimate \( \|u_r\|_p = O((1 - r)^{1/p-1}) \) as \( r \to 1 \) is sharp. The Alexiewicz norm estimates \( \|u_r\|_p \leq \|f\|_p \) \((0 \leq r < 1)\) and \( \|u_r - f\| \to 0 \) \((r \to 1)\) hold. These estimates lead to two uniqueness theorems for the Dirichlet problem in the unit disc with Henstock-Kurzweil integrable boundary data. There are similar growth estimates when \( u \) is in the harmonic Hardy space associated with the Alexiewicz norm and when \( f \) is of bounded variation.