Some Questions about Semisimple Lie Groups Originating in Matrix Theory

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Abstract. We generalize the well-known result that a square traceless complex matrix is unitarily similar to a matrix with zero diagonal to arbitrary connected semisimple complex Lie groups $G$ and their Lie algebras $\mathfrak{g}$ under the action of a maximal compact subgroup $K$ of $G$. We also introduce a natural partial order on $\mathfrak{g}$: $x \leq y$ if $f(K \cdot x) \subseteq f(K \cdot y)$ for all $f \in \mathfrak{g}^*$, the complex dual of $\mathfrak{g}$. This partial order is $K$-invariant and induces a partial order on the orbit space $\mathfrak{g}/K$. We prove that, under some restrictions on $\mathfrak{g}$, the set $f(K \cdot x)$ is star-shaped with respect to the origin.