Pure Discrete Spectrum for One-dimensional Substitution Systems of Pisot Type

Dedicated to Robert V. Moody on his 60-th birthday

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Abstract. We consider two dynamical systems associated with a substitution of Pisot type: the usual $\mathbb{Z}$-action on a sequence space, and the $\mathbb{R}$-action, which can be defined as a tiling dynamical system or as a suspension flow. We describe procedures for checking when these systems have pure discrete spectrum (the "balanced pairs algorithm" and the "overlap algorithm") and study the relation between them. In particular, we show that pure discrete spectrum for the $\mathbb{R}$-action implies pure discrete spectrum for the $\mathbb{Z}$-action, and obtain a partial result in the other direction. As a corollary, we prove pure discrete spectrum for every $\mathbb{R}$-action associated with a two-symbol substitution of Pisot type (this is conjectured for an arbitrary number of symbols).