Group Gradings on Matrix Algebras

Dedicated to the 60th birthday of Robert Moody

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Abstract. Let $\Phi$ be an algebraically closed field of characteristic zero, $G$ a finite, not necessarily abelian, group. Given a $G$-grading on the full matrix algebra $A = M_n(\Phi)$, we decompose $A$ as the tensor product of graded subalgebras $A = B \otimes C$, $B \cong M_p(\Phi)$ being a graded division algebra, while the grading of $C \cong M_q(\Phi)$ is determined by that of the vector space $\Phi^n$. Now the grading of $A$ is recovered from those of $A$ and $B$ using a canonical “induction” procedure.