Coordinatization Theorems
For Graded Algebras

Dedicated to Robert Moody on the occasion of his 60th birthday

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Abstract. In this paper we study simple associative algebras with finite $\mathbb{Z}$-gradings. This is done using a simple algebra $F_g$ that has been constructed in Morita theory from a bilinear form $g: U \times V \to A$ over a simple algebra $A$. We show that finite $\mathbb{Z}$-gradings on $F_g$ are in one to one correspondence with certain decompositions of the pair $(U, V)$. We also show that any simple algebra $R$ with finite $\mathbb{Z}$-grading is graded isomorphic to $F_g$ for some bilinear from $g: U \times V \to A$, where the grading on $F_g$ is determined by a decomposition of $(U, V)$ and the coordinate algebra $A$ is chosen as a simple ideal of the zero component $R_0$ of $R$. In order to prove these results we first prove similar results for simple algebras with Peirce gradings.