A Remark on the Moser-Aubin Inequality for Axially Symmetric Functions on the Sphere

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Abstract. Let $S_r$ be the collection of all axially symmetric functions $f$ in the Sobolev space $H^1(S^2)$ such that $\int_{S^2} x_i e^{2f(x)} d\omega(x)$ vanishes for $i = 1, 2, 3$. We prove that

$$\inf_{f \in S_r} \frac{1}{2} \int_{S^2} |\nabla f|^2 d\omega + 2 \int_{S^2} f d\omega - \log \int_{S^2} e^f d\omega > -\infty,$$

and that this infimum is attained. This complements recent work of Feldman, Froese, Ghoussoub and Gui on a conjecture of Chang and Yang concerning the Moser-Aubin inequality.

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