A Generalized Characterization of Commutators of Parabolic Singular Integrals

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Abstract. Let $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ and $\delta_\lambda x = (\lambda^{\alpha_1} x_1, \ldots, \lambda^{\alpha_n} x_n)$, where $\lambda > 0$ and $1 \leq \alpha_1 \leq \cdots \leq \alpha_n$. Denote $|\alpha| = \alpha_1 + \cdots + \alpha_n$. We characterize those functions $A(x)$ for which the parabolic Calderón commutator

$$T_A f(x) \equiv \text{p.v.} \int_{\mathbb{R}^n} K(x - y)[A(x) - A(y)] f(y) \, dy$$

is bounded on $L^2(\mathbb{R}^n)$, where $K(\delta_\lambda x) = \lambda^{-|\alpha| - 1} K(x)$, $K$ is smooth away from the origin and satisfies a certain cancellation property.