Smooth Maps and Real Algebraic Morphisms

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Abstract. Let $X$ be a compact nonsingular real algebraic variety and let $Y$ be either the blowup of $\mathbb{P}^n(\mathbb{R})$ along a linear subspace or a nonsingular hypersurface of $\mathbb{P}^m(\mathbb{R}) \times \mathbb{P}^n(\mathbb{R})$ of bidegree $(1, 1)$. It is proved that a $C^\infty$ map $f: X \to Y$ can be approximated by regular maps if and only if $f^*(H^1(Y, \mathbb{Z}/2)) \subseteq H^1_{\text{alg}}(X, \mathbb{Z}/2)$, where $H^1_{\text{alg}}(X, \mathbb{Z}/2)$ is the subgroup of $H^1(X, \mathbb{Z}/2)$ generated by the cohomology classes of algebraic hypersurfaces in $X$. This follows from another result on maps into generalized flag varieties.