Dow’s Principle and $Q$-Sets

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Abstract. A $Q$-set is a set of reals every subset of which is a relative $G_δ$. We investigate the combinatorics of $Q$-sets and discuss a question of Miller and Zhou on the size $q$ of the smallest set of reals which is not a $Q$-set. We show in particular that various natural lower bounds for $q$ are consistently strictly smaller than $q$.  

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