How the Roots of a Polynomial Vary with its Coefficients: A Local Quantitative Result

Bernard Beauzamy

Abstract. A well-known result, due to Ostrowski, states that if \( \|P - Q\|_2 < \varepsilon \), then the roots \((x_j)\) of \(P\) and \((y_j)\) of \(Q\) satisfy \( |x_j - y_j| \leq C n \varepsilon^{1/n} \), where \(n\) is the degree of \(P\) and \(Q\). Though there are cases where this estimate is sharp, it can still be made more precise in general, in two ways: first by using Bombieri's norm instead of the classical \(l_1\) or \(l_2\) norms, and second by taking into account the multiplicity of each root. For instance, if \(x\) is a simple root of \(P\), we show that \( |x - y| < C \varepsilon\) instead of \(\varepsilon^{1/n}\). The proof uses the properties of Bombieri's scalar product and Walsh Contraction Principle.