NEW CHARACTERIZATIONS OF THE REFLEXIVITY IN TERMS OF THE SET OF NORM ATTAINING FUNCTIONALS

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ABSTRACT. As a consequence of results due to Bourgain and Stegall, on a separable Banach space whose unit ball is not dentable, the set of norm attaining functionals has empty interior (in the norm topology). First we show that any Banach space can be renormed to fail this property. Then, our main positive result can be stated as follows: if a separable Banach space $X$ is very smooth or its bidual satisfies the $w^*$-Mazur intersection property, then either $X$ is reflexive or the set of norm attaining functionals has empty interior, hence the same result holds if $X$ has the Mazur intersection property and so, if the norm of $X$ is Fréchet differentiable. However, we prove that smoothness is not a sufficient condition for the same conclusion.