ON A PROBLEM OF RUBEL
CONCERNING THE SET OF FUNCTIONS SATISFYING
ALL THE ALGEBRAIC DIFFERENTIAL EQUATIONS
SATISFIED BY A GIVEN FUNCTION

JOHN SHACKELL

ABSTRACT. For two functions $f$ and $g$, define $g \ll f$ to mean that $g$ satisfies every algebraic differential equation over the constants satisfied by $f$. The order $\ll$ was introduced in one of a set of problems on algebraic differential equations given by the late Lee Rubel. Here we characterise the set of $g$ such that $g \ll f$, when $f$ is a given Liouvillian function.