ON THE SOLVABILITY OF A NEUMANN BOUNDARY VALUE PROBLEM AT RESONANCE

CHUNG-CHENG KUO

ABSTRACT. We study the existence of solutions of the semilinear equations (1) $\Delta u + g(x, u) = h$, $\frac{\partial u}{\partial n} = 0$ on $\partial \Omega$ in which the non-linearity $g$ may grow superlinearly in $u$ in one of directions $u \to \infty$ and $u \to -\infty$, and (2) $-\Delta u + g(x, u) = h$, $\frac{\partial u}{\partial n} = 0$ on $\partial \Omega$ in which the nonlinear term $g$ may grow superlinearly in $u$ as $|u| \to \infty$. The purpose of this paper is to obtain solvability theorems for (1) and (2) when the Landesman-Lazer condition does not hold. More precisely, we require that $h$ may satisfy $\int g^\gamma (x) dx < \int h(x) dx < \int g^\delta (x) dx$, where $\gamma, \delta$ are arbitrarily nonnegative constants, $g^\gamma (x) = \lim_{u \to \infty} \inf g(x, u)|u|^\gamma$ and $g^\delta (x) = \lim_{u \to -\infty} \sup g(x, u)|u|^\delta$. The proofs are based upon degree theoretic arguments.

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