QUANTUM DEFORMATIONS
OF SIMPLE LIE ALGEBRAS

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ABSTRACT. It is shown that every simple complex Lie algebra \( \mathfrak{g} \) admits a 1-parameter family \( \mathfrak{g}_q \) of deformations outside the category of Lie algebras. These deformations are derived from a tensor product decomposition for \( U_q(\mathfrak{g}) \)-modules; here \( U_q(\mathfrak{g}) \) is the quantized enveloping algebra of \( \mathfrak{g} \). From this it follows that the multiplication on \( \mathfrak{g}_q \) is \( U_q(\mathfrak{g}) \)-invariant. In the special case \( \mathfrak{g} = \mathfrak{sl}(2) \), the structure constants for the deformation \( \mathfrak{sl}(2)_q \) are obtained from the quantum Clebsch-Gordan formula applied to \( V(2)_q \otimes V(2)_q \); here \( V(2)_q \) is the simple 3-dimensional \( U_q(\mathfrak{sl}(2)) \)-module of highest weight \( q^2 \).

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