DERIVATIONS FROM TOTALLY ORDERED SEMIGROUP ALGEBRAS INTO THEIR DUALS

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ABSTRACT. For a well-behaved measure \( \mu \), on a locally compact totally ordered set \( X \), with continuous part \( \mu_c \), we make \( L^p(X, \mu_c) \) into a commutative Banach bimodule over the totally ordered semigroup algebra \( L^0(X, \mu) \), in such a way that the natural surjection from the algebra to the module is a bounded derivation. This gives rise to bounded derivations from \( L^p(X, \mu) \) into its dual module and in particular shows that if \( \mu_c \) is not identically zero then \( L^p(X, \mu) \) is not weakly amenable. We show that all bounded derivations from \( L^1(X, \mu) \) into its dual module arise in this way and also describe all bounded derivations from \( L^p(X, \mu) \) into its dual for \( 1 < p < \infty \) in the case that \( X \) is compact and \( \mu \) continuous.