

# On Dirichlet spaces with a class of superharmonic weights

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*Abstract.* In this paper, we investigate Dirichlet spaces  $\mathcal{D}_\mu$  with superharmonic weights induced by positive Borel measures  $\mu$  on the open unit disk. We establish the Alexander-Taylor-Ullman inequality for  $\mathcal{D}_\mu$  spaces and we characterize the cases where equality occurs. We define a class of weighted Hardy spaces  $H_\mu^2$  via the balayage of the measure  $\mu$ . We show that  $\mathcal{D}_\mu$  is equal to  $H_\mu^2$  if and only if  $\mu$  is a Carleson measure for  $\mathcal{D}_\mu$ . As an application, we obtain the reproducing kernel of  $\mathcal{D}_\mu$  when  $\mu$  is an infinite sum of point mass measures. We consider the boundary behavior and inner-outer factorization of functions in  $\mathcal{D}_\mu$ . We also characterize the boundedness and compactness of composition operators on  $\mathcal{D}_\mu$ .