Stability for the Brunn-Minkowski and Riesz rearrangement inequalities, with applications to Gaussian concentration and finite range non-local isoperimetry

Eric Carlen and Francesco Maggi

Abstract. We provide a simple, general argument to obtain improvements of concentration-type inequalities starting from improvements of their corresponding isoperimetric-type inequalities. We apply this argument to obtain robust improvements of the Brunn-Minkowski inequality (for Minkowski sums between generic sets and convex sets) and of the Gaussian concentration inequality. The former inequality is then used to obtain a robust improvement of the Riesz rearrangement inequality under certain natural conditions. These conditions are compatible with the applications to a finite-range nonlocal isoperimetric problem arising in statistical mechanics.