The seven dimensional perfect Delaunay polytopes and Delaunay simplices
Mathieu Dutour Sikirić

Abstract. For a lattice $L$ of $\mathbb{R}^n$, a sphere $S(c, r)$ of center $c$ and radius $r$ is called empty if for any $v \in L$ we have $\|v - c\| \geq r$. Then the set $S(c, r) \cap L$ is the vertex set of a Delaunay polytope $P = \text{conv}(S(c, r) \cap L)$. A Delaunay polytope is called perfect if any affine transformation $\phi$ such that $\phi(P)$ is a Delaunay polytope is necessarily an isometry of the space composed with an homothety.

Perfect Delaunay polytopes are remarkable structure that exist only if $n = 1$ or $n \geq 6$ and they have shown up recently in covering maxima studies. Here we give a general algorithm for their enumeration that relies on the Erdahl cone. We apply this algorithm in dimension 7 which allow us to find that there are only two perfect Delaunay polytopes: $3_{21}$ which is a Delaunay polytope in the root lattice $E_7$ and the Erdahl Rybnikov polytope.

We then use this classification in order to get the list of all types Delaunay simplices in dimension 7 and found 11 types.