Number of right ideals and a $q$-analogue of indecomposable permutations
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Abstract. We prove that the number of right ideals of codimension $n$ in the algebra of noncommutative Laurent polynomials in two variables over the finite field $\mathbb{F}_q$ is equal to $(q - 1)^{n+1} q^{(n+1)(n-2)/2} \sum_{\theta} q^{\text{inv}(\theta)}$, where the sum is over all indecomposable permutations in $S_{n+1}$ and where $\text{inv}(\theta)$ stands for the number of inversions of $\theta$. 