Constrained approximation with Jacobi weights
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Abstract. In this paper, we prove that, for \( \ell = 1 \) or \( \ell = 2 \), the rate of best \( \ell \)-monotone polynomial approximation in the \( L_p \) norm \( (1 \leq p \leq \infty) \) weighted by the Jacobi weight
\[
w_{\alpha, \beta}(x) := (1 + x)^{\alpha} (1 - x)^{\beta}
\]
with \( \alpha, \beta > -1/p \) if \( p < \infty \), or \( \alpha, \beta \geq 0 \) if \( p = \infty \), is bounded by an appropriate \( (\ell + 1) \)st modulus of smoothness with the same weight, and that this rate cannot be bounded by the \( (\ell + 2) \)nd modulus. Related results on constrained weighted spline approximation and applications of our estimates are also given.