Pathological phenomena in Denjoy-Carleman classes

Ethan Y. Jaffe

Abstract. Let $\mathcal{C}^M$ denote a Denjoy-Carleman class of $\mathcal{C}^\infty$ functions (for a given logarithmically-convex sequence $M = (M_n)$). We construct: (1) a function in $\mathcal{C}^M((-1, 1))$ which is nowhere in any smaller class; (2) a function on $\mathbb{R}$ which is formally $\mathcal{C}^M$ at every point, but not in $\mathcal{C}^M(\mathbb{R})$; (3) (under the assumption of quasianalyticity) a smooth function on $\mathbb{R}^p$ ($p \geq 2$) which is $\mathcal{C}^M$ on every $\mathcal{C}^M$ curve, but not in $\mathcal{C}^M(\mathbb{R}^p)$. 