Abelian Surfaces with an Automorphism and Quaternionic Multiplication

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Abstract. We construct one dimensional families of Abelian surfaces with quaternionic multiplication which also have an automorphism of order three or four. Using Barth’s description of the moduli space of $(2, 4)$-polarized Abelian surfaces, we find the Shimura curve parametrizing these Abelian surfaces in a specific case. We explicitly relate these surfaces to the Jacobians of genus two curves studied by Hashimoto and Murabayashi. We also describe a (Humbert) surface in Barth’s moduli space which parametrizes Abelian surfaces with real multiplication by $\mathbb{Z}[\sqrt{2}]$. 