Rotation algebras and the Exel trace formula

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Abstract. We found that if $u$ and $v$ are any two unitaries in a unital $C^*$-algebra with $\|uv - vu\| < 2$ and $uvu^*v^*$ commutes with $u$ and $v$, then the $C^*$-subalgebra $A_{u,v}$ generated by $u$ and $v$ is isomorphic to a quotient of some rotation algebra $A_\theta$ provided that $A_{u,v}$ has a unique tracial state. We also found that the Exel trace formula holds in any unital $C^*$-algebra. Let $\theta \in (-1/2, 1/2)$ be a real number. We prove the following: For any $\epsilon > 0$, there exists $\delta > 0$ satisfying the following: if $u$ and $v$ are two unitaries in any unital simple $C^*$-algebra $A$ with tracial rank zero such that $\|uv - e^{2\pi i \theta}vu\| < \delta$ and $\frac{1}{2\pi i} \tau(\log(uvu^*v^*)) = \theta$, for all tracial state $\tau$ of $A$, then there exists a pair of unitaries $\tilde{u}$ and $\tilde{v}$ in $A$ such that

$$\tilde{u}\tilde{v} = e^{2\pi i \theta} \tilde{v}\tilde{u}, \quad \|u - \tilde{u}\| < \epsilon \quad \text{and} \quad \|v - \tilde{v}\| < \epsilon.$$