Abstract. We provide some new local obstructions to approximating tropical curves in smooth tropical surfaces. These obstructions are based on a relation between tropical and complex intersection theories which is also established here. We give two applications of the methods developed in this paper. First, we classify all locally irreducible approximable 3-valent fan tropical curves in a fan tropical plane. Secondly, we prove that a generic non-singular tropical surface in tropical projective 3-space contains finitely many approximable tropical lines if it is of degree 3, and contains no approximable tropical lines if it is of degree 4 or more.