On a sumset conjecture of Erdős

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Abstract. Erdős conjectured that for any set $A \subseteq \mathbb{N}$ with positive lower asymptotic density, there are infinite sets $B, C \subseteq \mathbb{N}$ such that $B+C \subseteq A$. We verify Erdős' conjecture in the case that A has *Banach* density exceeding $\frac{1}{2}$. As a consequence, we prove that, for $A \subseteq \mathbb{N}$ with positive Banach density (a much weaker assumption than positive lower density), we can find infinite $B, C \subseteq \mathbb{N}$ such that $B + C$ is contained in the union of A and a translate of A. Both of the aforementioned results are generalized to arbitrary countable amenable groups. We also provide a positive solution to Erdős' conjecture for subsets of the natural numbers that are *pseudorandom*.