

# On Varieties of Lie Algebras of Maximal Class

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*Abstract.* We study complex projective varieties that parametrize (finite-dimensional) filiform Lie algebras over  $\mathbb{C}$ , using equations derived by Millionshchikov. In the infinite-dimensional case we concentrate our attention on  $\mathbb{N}$ -graded Lie algebras of maximal class. As shown by A. Fialowski there are only three isomorphism types of  $\mathbb{N}$ -graded Lie algebras  $L = \bigoplus_{i=1}^{\infty} L_i$  of maximal class generated by  $L_1$  and  $L_2$ ,  $L = \langle L_1, L_2 \rangle$ . Vergne described the structure of these algebras with the property  $L = \langle L_1 \rangle$ . In this paper we study those generated by the first and  $q$ -th components where  $q > 2$ ,  $L = \langle L_1, L_q \rangle$ . Under some technical condition, there can only be one isomorphism type of such algebras. For  $q = 3$  we fully classify them. This gives a partial answer to a question posed by Millionshchikov.