Abstract. The paper studies the $K$-theoretic invariants of the crossed product $C^*$-algebras associated with an important family of homeomorphisms of the tori $\mathbb{T}^n$ called Furstenberg transformations. Using the Pimsner-Voiculescu theorem, we prove that given $n$, the $K$-groups of those crossed products, whose corresponding $n \times n$ integer matrices are unipotent of maximal degree, always have the same rank $a_n$. We show using the theory developed here that a claim made in the literature about the torsion subgroups of these $K$-groups is false. Using the representation theory of the simple Lie algebra $\mathfrak{sl}(2, \mathbb{C})$, we show that, remarkably, $a_n$ has a combinatorial significance. For example, every $a_{2n+1}$ is just the number of ways that 0 can be represented as a sum of integers between $-n$ and $n$ (with no repetitions). By adapting an argument of van Lint (in which he answered a question of Erdős), a simple, explicit formula for the asymptotic behavior of the sequence $\{a_n\}$ is given. Finally, we describe the order structure of the $K_0$-groups of an important class of Furstenberg crossed products, obtaining their complete Elliott invariant using classification results of H. Lin and N. C. Phillips.