Galois Representations Over Fields of Moduli and Rational Points on Shimura Curves

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Abstract. The purpose of this note is introducing a method for proving the existence of no rational points on a coarse moduli space X of abelian varieties over a given number field K, in cases where the moduli problem is not fine and points in $X(K)$ may not be represented by an abelian variety (with additional structure) admitting a model over the field K. This is typically the case when the abelian varieties that are being classified have even dimension. The main idea, inspired on the work of Ellenberg and Skinner on the modularity of $\mathbb{Q}$-curves, is that to a point $P = [A] \in X(K)$ represented by an abelian variety $A/\bar{K}$ one may still attach a Galois representation of $\mathrm{Gal}(\bar{K}/K)$ with values in the quotient group $\mathrm{GL}(T_\ell(A))/\mathrm{Aut}(A)$, provided $\mathrm{Aut}(A)$ lies in the centre of $\mathrm{GL}(T_\ell(A))$. We exemplify our method in the cases where X is a Shimura curve over an imaginary quadratic field or an Atkin-Lehner quotient over $\mathbb{Q}$.