3-torsion in the Homology of Complexes of Graphs of Bounded Degree
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Abstract. For $\delta \geq 1$ and $n \geq 1$, consider the simplicial complex of graphs on $n$ vertices in which each vertex has degree at most $\delta$; we identify a given graph with its edge set and admit one loop at each vertex. This complex is of some importance in the theory of semigroup algebras. When $\delta = 1$, we obtain the matching complex, for which it is known that there is 3-torsion in degree $d$ of the homology whenever $\frac{n-4}{3} \leq d \leq \frac{n-6}{2}$. This paper establishes similar bounds for $\delta \geq 2$. Specifically, there is 3-torsion in degree $d$ whenever $\frac{(3\delta-1)n-8}{6} \leq d \leq \frac{\delta(n-1)-4}{2}$. The procedure for detecting torsion is to construct an explicit cycle $z$ that is easily seen to have the property that $3z$ is a boundary. Defining a homomorphism that sends $z$ to a non-boundary element in the chain complex of a certain matching complex, we obtain that $z$ itself is a non-boundary. In particular, the homology class of $z$ has order 3.