The Ranks of the Homotopy Groups of a Finite dimensional Complex
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Abstract. Let $X$ be an $n$-dimensional, finite, simply connected CW complex and set
$$\alpha_X = \limsup_i \frac{\log \text{rank} \pi_i(X)}{i}. $$
When $0 < \alpha_X < \infty$, we give upper and lower bound for
$$\sum_{i=k+2}^{k+n} \text{rank} \pi_i(X)$$
for $k$ sufficiently large. We show also for any $r$ that $\alpha_X$ can be estimated from the integers $\text{rk} \pi_i(X), i \leq nr$ with an error bound depending explicitly on $r$. 