Analysis of the Brylinski-Kostant Model for Spherical Minimal Representations
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Abstract. We revisit with another view point the construction by R. Brylinski and B. Kostant of minimal representations of simple Lie groups. We start from a pair \((V, Q)\), where \(V\) is a complex vector space and \(Q\) a homogeneous polynomial of degree 4 on \(V\). The manifold \(\Xi\) is an orbit of a covering of Conf\((V, Q)\), the conformal group of the pair \((V, Q)\), in a finite dimensional representation space. By a generalized Kantor-Koecher-Tits construction we obtain a complex simple Lie algebra \(\mathfrak{g}\), and furthermore a real form \(\mathfrak{g}_\mathbb{R}\). The connected and simply connected Lie group \(G_\mathbb{R}\) with \(\text{Lie}(G_\mathbb{R}) = \mathfrak{g}_\mathbb{R}\) acts unitarily on a Hilbert space of holomorphic functions defined on the manifold \(\Xi\).