Densities of Short Uniform Random Walks
Jonathan M. Borwein, Armin Straub, James Wan, and Wadim Zudilin

Abstract. We study the densities of uniform random walks in the plane. A special focus is on the case of short walks with three or four steps and less completely those with five steps. As one of the main results, we obtain a hypergeometric representation of the density for four steps, which complements the classical elliptic representation in the case of three steps. It appears unrealistic to expect similar results for more than five steps. New results are also presented concerning the moments of uniform random walks and, in particular, their derivatives. Relations with Mahler measures are discussed.