Boundedness of Calderón–Zygmund Operators on Non-homogeneous Metric Measure Spaces
Tuomas Hytönen, Suile Liu, Dachun Yang, and Dongdong Yang

Abstract. Let \((X, d, \mu)\) be a separable metric measure space satisfying the known upper doubling condition, the geometrical doubling condition, and the non-atomic condition that \(\mu(\{x\}) = 0\) for all \(x \in X\). In this paper, we show that the boundedness of a Calderón–Zygmund operator \(T\) on \(L^2(\mu)\) is equivalent to that of \(T\) on \(L^p(\mu)\) for some \(p \in (1, \infty)\), and that of \(T\) from \(L^1(\mu)\) to \(L^{1,\infty}(\mu)\). As an application, we prove that if \(T\) is a Calderón–Zygmund operator bounded on \(L^2(\mu)\), then its maximal operator is bounded on \(L^p(\mu)\) for all \(p \in (1, \infty)\) and from the space of all complex-valued Borel measures on \(X\) to \(L^{1,\infty}(\mu)\). All these results generalize the corresponding results of Nazarov et al. on metric spaces with measures satisfying the so-called polynomial growth condition.