Abstract. We introduce the fundamental group $\mathcal{F}(A)$ of a simple $\sigma$-unital $C^*$-algebra $A$ with unique (up to scalar multiple) densely defined lower semicontinuous trace. This is a generalization of Fundamental Group of Simple $C^*$-algebras with Unique Trace I and II by Nawata and Watatani. Our definition in this paper makes sense for stably projectionless $C^*$-algebras. We show that there exist separable stably projectionless $C^*$-algebras such that their fundamental groups are equal to $\mathbb{R}_+^\infty$ by using the classification theorem of Razak and Tsang. This is a contrast to the unital case in Nawata and Watatani. This study is motivated by the work of Kishimoto and Kumjian.

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