



Compactness of Commutators for Singular Integrals on Morrey Spaces

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Abstract. In this paper we characterize the compactness of the commutator $[b, T]$ for the singular integral operator on the Morrey spaces $L^{p,\lambda}(\mathbb{R}^n)$. More precisely, we prove that if $b \in \text{VMO}(\mathbb{R}^n)$, the $\text{BMO}(\mathbb{R}^n)$ -closure of $C_c^\infty(\mathbb{R}^n)$, then $[b, T]$ is a compact operator on the Morrey spaces $L^{p,\lambda}(\mathbb{R}^n)$ for $1 < p < \infty$ and $0 < \lambda < n$. Conversely, if $b \in \text{BMO}(\mathbb{R}^n)$ and $[b, T]$ is a compact operator on the $L^{p,\lambda}(\mathbb{R}^n)$ for some p ($1 < p < \infty$), then $b \in \text{VMO}(\mathbb{R}^n)$. Moreover, the boundedness of a rough singular integral operator T and its commutator $[b, T]$ on $L^{p,\lambda}(\mathbb{R}^n)$ are also given. We obtain a sufficient condition for a subset in Morrey space to be a strongly pre-compact set, which has interest in its own right.

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