Critical Points and Resonance of Hyperplane Arrangements

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Abstract. If \( \Phi_\lambda \) is a master function corresponding to a hyperplane arrangement \( A \) and a collection of weights \( \lambda \), we investigate the relationship between the critical set of \( \Phi_\lambda \), the variety defined by the vanishing of the one-form \( \omega_\lambda = d \log \Phi_\lambda \), and the resonance of \( \lambda \). For arrangements satisfying certain conditions, we show that if \( \lambda \) is resonant in dimension \( p \), then the critical set of \( \Phi_\lambda \) has codimension at most \( p \). These include all free arrangements and all rank 3 arrangements.

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